A new general type of solitary wave of a (2+1)-dimensional system

2005 J. Phys. A: Math. Gen. 38 4375

(http://iopscience.iop.org/0305-4470/38/20/006)

J. Phys. A: Math. Gen. **38** (2005) 4375–4389 doi:10.1088/0305-4470/38/20/006

A new general type of solitary wave of a (2+1)-dimensional system

Cheng-Lin Bai and Hong Zhao

School of Physics Science and Information Engineering, Liaocheng University, Shandong 252059, People's Republic of China

E-mail: lcced_bcl@hotmail.com and lcced_bcl@lctu.edu.cn

Received 9 December 2004, in final form 24 March 2005
Published 3 May 2005
Online at stacks.iop.org/JPhysA/38/4375

Abstract

Using the extended homogeneous balance method (EHBM) and variable separation approach (VSA), an exact variable separation excitation of a (2+1)-dimensional system is derived. Based on the derived excitation, a new general type of solitary wave, i.e., semifolded solitary waves and semifoldons (SFSWs), is defined and studied. We investigate the behaviours of the interactions for the semifolded localized structures and find that the interactions possess some novel and interesting features.

PACS numbers: 02.30.Jr, 05.45.Yv

1. Introduction

In non-linear science, soliton theory plays an essential role and has been applied in almost all natural sciences, especially in all physics branches such as condensed matter physics, field theory, fluid dynamics, plasma physics, optics, etc [1]. Most of the previous studies on soliton theory, especially in higher dimensions, have been restricted to the single-valued situations, such as dromion, compacton, peakon and their interactions. However, in various cases, real natural phenomena are too intricate to describe only by virtue of single-valued functions. For instance, in nature, there exist very complicated folded phenomena such as the folded protein [2], folded brain and skin surface and many other kinds of folded biological systems [3]. The simplest multi-valued (folded) waves may be the bubbles on (or under) a fluid surface. Various ocean waves are really folded waves too. In [4–6], the authors discussed some simpler cases of multiple-valued solitary waves (folded in all directions). However, nature is extremely colourful and may exhibit quite complicated structures. For example, some structures such as ocean waves may fold in one direction, say x, and localize in a usual single-valued way in another direction, say y. Of course, at the present stage, it is impossible to make satisfactory analytic descriptions for such complicated folded natural phenomena. But it is still worth starting with some simpler cases. Similar to the single-valued case, the primary question we should (and we can) ask is: are there any stable multi-valued (folded in one direction) localized excitations? For convenience, we define the multi-valued localized excitations as semifolded solitary waves (SFSWs). Furthermore, if the interactions among the semifolded solitary waves are completely elastic, we call them semifoldons. Within our knowledge, studying the semifolded localized excitations in higher dimensional physical models is still open. Meanwhile, how to find new localized excitations is one of the most fundamental and significant studies in non-linear and theoretical physics. Motivated by these reasons, we take the (2+1)-dimensional generalized Nizhnik–Novikov–Veselov (GNNV) system

$$u_t + au_{xxx} + bu_{yyy} + cu_x + du_y - 3a(uv)_x - 3b(uw)_y = 0, \tag{1a}$$

$$u_x = v_y, \tag{1b}$$

$$u_y = w_x, \tag{1c}$$

where a, b, c and d are arbitrary constants, as a concrete example. For $c = d = 0$, the (2+1)-dimensional GNNV system will be degenerated to the usual two-dimensional NNV system, which is an only known isotropic Lax integrable extension of the classical (1+1)-dimensional shallow water wave KdV model. Boiti *et al* [7] solved the GNNV system by using the inverse scattering transformation (IST), and Radha and Lakshmanan [8] constructed its multi-dromion solutions. By using the variable separation approach (VSA) based on the extended homogeneous balance method (EHBM) [9], we obtain a general variable-separated solution. This solution turns out to be a quite 'universal' formula, and is valid for suitable fields or potentials of various (2+1)-dimensional physically interesting integrable models, including the Davey–Stewartson (DS) equation, the dispersive long wave equation (DLWE) [4], the Broer–Kaup (BK) system [4, 9], the higher order Broer–Kaup system [5], the Nizhnik–Novikov–Vesselov (NNV) system, the ANNV (asymmetric NNV) equation and so on [4]. In principle, following the general ideas introduced in [10], one could investigate the stability properties of the solutions presented in this paper and their relevance as asymptotic states for suitable initial boundary value problems. However, here, we study only the interaction behaviour among the localized solutions by studying the asymptotic property of the 'universal' formula because these formulae are valid for more than one system.

The paper is organized as follows. In section 2, we apply a variable-separated approach based on the extended homogeneous balance method (VSA on EHBM) to solve the (2+1)-dimensional GNNV and obtain its exact excitation. Section 3 is devoted to investigating the interaction properties both for the semifoldons and between single-valued and semifolded localized excitations. A brief discussion and summary is given in the last section.

2. Variable-separated solutions for (2+1)-dimensional GNNV equation

According to the EHBM, let

$$u = f(\varphi)_{xy} + u_0, \tag{2a}$$

$$v = g(\varphi)_{xx} + v_0, \tag{2b}$$

$$w = h(\varphi)_{yy} + w_0, \tag{2c}$$

where $f(\varphi)$, $g(\varphi)$ and $h(\varphi)$ are functions of one argument φ only, $\varphi = \varphi(x, y, t)$, and $\{u_0, v_0, w_0\}$ are arbitrary known seed solutions of the GNNV equation.

Substituting equations (2*a*)–(2*c*) into equations (1*a*)–(1*c*), respectively, and collecting all homogeneous terms in the partial derivatives of $\varphi(x, y, t)$, we obtain

$$\begin{aligned} u_t + au_{xxx} + bu_{yyy} + cu_x + du_y - 3a(uv)_x - 3b(uw)_y \\ = (f^{(5)} - 3f''g^{(3)} - 3f^{(3)}g'')a\varphi_x^4\varphi_y + (f^{(5)} - 3f''h^{(3)} - 3f^{(3)}h'')b\varphi_x\varphi_y^4 \\ + \text{lower power terms of the derivatives of } \varphi(x, y, t) \text{ with respect to } x, y \\ \text{and } t = 0, \end{aligned} \tag{3a}$$

$$(f^{(3)} - g^{(3)})\varphi_x^2\varphi_y + (f'' - g'')\varphi_{xx}\varphi_y + 2(f'' - g'')\varphi_x\varphi_{xy} + (f' - g')\varphi_{xxy} + u_{0x} - v_{0y} = 0, \tag{3b}$$

$$(f^{(3)} - h^{(3)})\varphi_x\varphi_y^2 + (f'' - h'')\varphi_x\varphi_{yy} + 2(f'' - h'')\varphi_y\varphi_{xy} + (f' - g')\varphi_{xyy} + u_{0y} - w_{0x} = 0. \tag{3c}$$

For simplicity, in equations (3*b*) and (3*c*), we take the special solution as

$$f = g = h, \qquad u_0 = 0, \qquad v_0 = v_0(x, t), \qquad w_0 = w_0(y, t). \tag{4}$$

Now we can simplify equation (3*a*). First, setting the coefficients of the terms with $a\varphi_x^4\varphi_y$ and $b\varphi_y^4\varphi_x$ to zero, we obtain an ordinary differential equation for function $f(\varphi)$:

$$f^{(5)} - 6f''f^{(3)} = 0, \tag{5}$$

which indicates that the non-linear terms and the second-order derivative terms appearing in equation (1) have been partially balanced. This is why we assume that the solutions of equation (1) are of the form of equations (2).

The following special solution exists for equation (5):

$$f = -2\ln\varphi. \tag{6}$$

Thereby

$$f'f'' = f^{(3)}, \qquad f'^2 = 2f^{(2)}, \qquad f''^2 = \left(\tfrac{1}{3}\right)f^{(4)}, \qquad f'f^{(3)} = \left(\tfrac{2}{3}\right)f^{(4)}.$$

Using these results, expression (3*a*) can be simplified as

$$\begin{aligned} \big(\varphi_x\varphi_y(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y) \\ + 3\big(a\varphi_x\varphi_{xx}\varphi_{xy} + b\varphi_y\varphi_{yy}\varphi_{xy} - a\varphi_x^2\varphi_{xxy} - b\varphi_y^2\varphi_{xyy}\big)\big)f^{(3)} \\ + (\varphi_y(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y)_x \\ + (\varphi_x(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y)_y \\ + \varphi_{xy}(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y) \\ + 3(-a\varphi_{xxx}\varphi_{xy} + a\varphi_x\varphi_{xxxy} - b\varphi_{xy}\varphi_{yyy} + b\varphi_y\varphi_{xyyy}))f'' \\ \times (\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y)_{xy}f' = 0. \end{aligned} \tag{7}$$

Setting the coefficients of $f^{(3)}$, f'' and f' in equation (7) to zero yields a set of equations for $\varphi(x, y, t)$:

$$\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} + c\varphi_x + d\varphi_y - 3av_0\varphi_x - 3bw_0\varphi_y = 0, \tag{8a}$$

$$a\varphi_x\varphi_{xx}\varphi_{xy} + b\varphi_y\varphi_{yy}\varphi_{xy} - a\varphi_x^2\varphi_{xxy} - b\varphi_y^2\varphi_{xyy} = 0, \tag{8b}$$

$$-a\varphi_{xxx}\varphi_{xy} + a\varphi_x\varphi_{xxxy} - b\varphi_{xy}\varphi_{yyy} + b\varphi_y\varphi_{xyyy} = 0. \tag{8c}$$

Because $v_0(x, t)$ and $w_0(y, t)$ are arbitrary functions of variables $\{x, t\}$ and $\{y, t\}$, respectively, in equations (8) we can select an appropriate variable-separated hypothesis for the function φ as follows:

$$\varphi(x, y, t) = a_0 + a_1 p(x, t) + a_2 q(y, t) + a_3 p(x, t) q(y, t), \tag{9}$$

where $p(x, t)$ is an arbitrary function of variables $\{x, t\}$, $q(y, t)$ is an arbitrary function of variables $\{y, t\}$, and a_0, a_1, a_2 and a_3 are four arbitrary constants. Substituting equations (6) and (9) into equations (2), along with equations (8), and carrying out some careful and tedious calculations, then the corresponding excitations for the (2+1)-dimensional GNNV system yield

$$u(x, y, t) = \frac{2(a_3 a_0 - a_1 a_2) p_x q_y}{(a_0 + a_1 p + a_2 q + a_3 pq)^2}, \tag{10a}$$

$$v(x, y, t) = \frac{2(a_1 + a_3 q)^2 p_x^2}{(a_0 + a_1 p + a_2 q + a_3 pq)^2} - \frac{2(a_1 + a_3 q) p_{xx}}{a_0 + a_1 p + a_2 q + a_3 pq} + \frac{p_t + a p_{xxx} + c p_x}{3 a p_x}, \tag{10b}$$

$$w(x, y, t) = \frac{2(a_2 + a_3 p)^2 q_y^2}{(a_0 + a_1 p + a_2 q + a_3 pq)^2} - \frac{2(a_1 + a_3 p) q_{yy}}{a_0 + a_1 p + a_2 q + a_3 pq} + \frac{q_t + b q_{yyy} + d q_y}{3 a q_y} \tag{10c}$$

with two arbitrary functions $p(x, t)$ and $q(y, t)$.

3. Some novel localized structures for a (2+1)-dimensional system

It is interesting to mention that by slightly scalar transformation, the expression (10*a*) is valid for many (2+1)-dimensional models, such as the DS equation, NNV system, ANNV equation and the BK equation, etc. Therefore, we can call the expression (10*a*) as a common field quantity. Moreover, because of the arbitrariness of the functions p and q included in (10*a*), the quantity u possesses quite rich structures. For instance, if we select the functions p and q appropriately, we can obtain many kinds of localized solutions, such as the multi-solitoff solutions, multi-dromion and dromion lattice solutions, multiple ring soliton solutions, peakons, compactons and so on [4, 5]. The properties of peakon–peakon, dromion–dromion, compacton–compacton and foldon–foldon interactions were discussed in [4–6, 9]. In [12], Bai *et al* investigate the interactions among different types of solitary waves such as peakons, dromions and compactons both analytically and graphically. Now we pay attention to the semifolded localized structures and interactions of single-valued and semifolded localized excitations. In order to discuss the interaction property of the localized excitations related to the physical quantity (10*a*), we first study the asymptotic behaviours of the localized excitations produced from (10*a*) when $t \to \pm\infty$.

3.1. Asymptotic behaviours of the localized excitations produced from (10a) [11, 12]

In general, if the function p and q are selected as multi-localized solitonic excitations with

$$p|_{t\to\mp\infty} = \sum_{i=1}^{M} p_i^{\mp}, \qquad p_i^{\mp} \equiv p_i\left(x - c_i t + \delta_i^{\mp}\right), \tag{11}$$

$$q|_{t\to\mp\infty} = \sum_{j=1}^{N} q_j^{\mp}, \qquad q_j^{\mp} \equiv q_j\left(y - C_j t + \Delta_j^{\mp}\right), \tag{12}$$

where $\{p_i, q_j\}\,\forall i$ and j are localized functions, then the physical quantity u expressed by equation (10a) delivers $M \times N$ (2+1)-dimensional localized excitations with the asymptotic behaviour

$$u|_{t\to\mp\infty} \to \sum_{i=1}^{M}\sum_{j=1}^{N}\left\{\frac{2(a_3a_0 - a_1a_2)p_{ix}^{\mp}q_{jy}^{\mp}}{\left(a_0 + a_1\left(p_i^{\mp} + P_i^{\mp}\right) + a_2\left(q_j^{\mp} + Q_j^{\mp}\right) + a_3\left(p_i^{\mp} + P_i^{\mp}\right)\left(q_j^{\mp} + Q_j^{\mp}\right)\right)^2}\right\} \tag{13}$$

where

$$P_i^{\mp} = \sum_{j<i} p_j(\mp\infty) + \sum_{j>i} p_j(\pm\infty), \tag{14}$$

$$Q_i^{\mp} = \sum_{j<i} q_j(\mp\infty) + \sum_{j>i} q_j(\pm\infty). \tag{15}$$

In the above, it has been assumed, without loss of generality, that $C_i > C_j$ and $c_i > c_j$ if $i > j$.

From the asymptotic result (13), we know that (i) the *ij*th localized excitation u_{ij} is a travelling wave moving with the velocity c_i along the positive ($c_i > 0$) or negative ($c_i < 0$) x-direction, and C_j along the positive ($C_j > 0$) or negative ($C_j < 0$) y-direction; (ii) the properties of the *ij*th localized excitation u_{ij} is only determined by p_i of equation (11) and q_j of equation (12); (iii) the shape of the *ij*th localized excitation u_{ij} will be changed if

$$P_i^+ \neq P_i^-, \tag{16a}$$

and (or)

$$Q_j^+ \neq Q_j^-, \tag{16b}$$

however, it will preserve its shape during the interaction if

$$P_i^+ = P_i^-, \qquad Q_j^+ = Q_j^-; \tag{17}$$

(iv) the phase shift of the *ij*th localized excitation u_{ij} reads

$$\delta_i^+ - \delta_i^- \tag{18}$$

in the x-direction and

$$\Delta_j^+ - \Delta_j^- \tag{19}$$

in the y-direction.

The above discussions demonstrate that multiple localized solitonic excitations for the universal quantity u can be constructed without difficulty via the (1+1)-dimensional localized excitations with the properties (11), (12), (16) and (17). As a matter of fact, any localized solutions (or their derivatives) with completely elastic (or not completely elastic or completely inelastic) interaction behaviours of any known (1+1)-dimensional integrable models can be utilized to construct (2+1)-dimensional localized solitonic solutions with completely elastic ($P_i^+ = P_i^-$, $Q_j^+ = Q_j^-$ for all i, j) (or not completely elastic or completely inelastic ($P_i^+ \neq P_i^-$, $Q_j^+ \neq Q_j^-$ for at least for one of i, j)) interaction properties. However, to the best of our knowledge, the interactions among semifoldons, peakon, dromions and compactons have not been reported previously in the literature. In order to see the interaction behaviours among them more directly and visually, we investigate some special examples by fixing the arbitrary functions p and q in equation (10a). For convenience, we set $a_0 = a_1 = -a_2 = 1$, $a_3 = 0.2$ in equation (10a) in the following discussion.

3.2. Completely elastic interactions

Now, we discuss some new coherent structures for the physical quantity u, and focus our attention on some (2+1)-dimensional semifolded localized structures, which may exist in certain situations, when the function q is t-independent and p is selected via the relations

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \qquad p = \int^{\xi} p_x x_\xi \, \mathrm{d}\xi, \tag{20}$$

where U_i and X_i are localized excitations with the properties $U_i(\pm\infty) = 0$, $X_i(\pm\infty) = \text{const}$. From equation (20), one can know that ξ may be a multi-valued function in some suitable regions of x by selecting the functions X_i appropriately. Therefore, the function p_x, which is obviously an interaction solution of M localized excitations because of the property $\xi|_{x\to\infty} \to \infty$, may be a multi-valued function of x in these areas, though it is a single-valued function of ξ. Actually, most of the known multi-loop solutions are a special situation of equation (20). In general terms, if the functions p or q are taken as multiple localized excitations that possess the phase shifts of (1+1)-dimensional models then the (2+1)-dimensional localized excitations involving representation (10a) inherit phase shifts structures. As simple choices for the functions p and q, one can take

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{21}$$

$$q = 1 + \sum_{j=1}^{N} \exp[k_j(y + \beta_j t) + y_{0j}], \tag{22}$$

where k_j, β_j, w_i and y_{0j} are arbitrary constants and M, N are positive integers. If we take the concrete forms of p and q as follows:

$$p_x = \tfrac{4}{5}\,\mathrm{sech}^2(\xi) + \tfrac{1}{2}\,\mathrm{sech}^2(\xi - 0.3t), \qquad x = \xi - 1.5\tanh(\xi) - 1.5\tanh(\xi - 0.3t), \tag{23}$$

$$q = 1 + \exp(y), \tag{24}$$

then we successfully construct semifolded localized excitations that possess phase shifts for the physical quantity u depicted in figure 1. From figure 1, we can see that the two semifolded localized excitations possess novel properties, which fold in the y-direction, and localize in a usual single-valued way in the x-direction. Moreover, one can find that the interaction between the two semifolded localized excitations (semifoldons) is completely elastic, which is very similar to the completely elastic collisions between two classical particles, since the velocity of one of the localized structures has set to be zero and there are still phase shifts for the two semifolded localized excitations. To see more carefully, one can easily find that the position located by the large static localized structure is altered from about $x = -1.5$ to $x = 1.5$ and its shape is completely preserved after interaction.

Along the same line of argument and performing a similar analysis, when p and q are taken as the following forms:

$$p_x = \sum_{j=1}^{M} U_j(\xi + w_j t), \qquad x = \xi + \sum_{j=1}^{M} X_j(\xi + w_j t), \tag{25}$$

$$q = \begin{cases} a_0, & y + \beta_i t \leqslant y_{0i} - \frac{\pi}{2k_i} \\ a_0 + \sum_{i=1}^{N} (b_i \sin(k_i(y + \beta_i t - y_{0i})) + b_i), & y_{0i} - \frac{\pi}{2k_i} < y + \beta_i t \leqslant y_{0i} + \frac{\pi}{2k_i} \\ a_0 + \sum_{i=1}^{M} 2b_i, & y + \beta_i t > y_{0i} + \frac{\pi}{2k_i}, \end{cases} \tag{26}$$

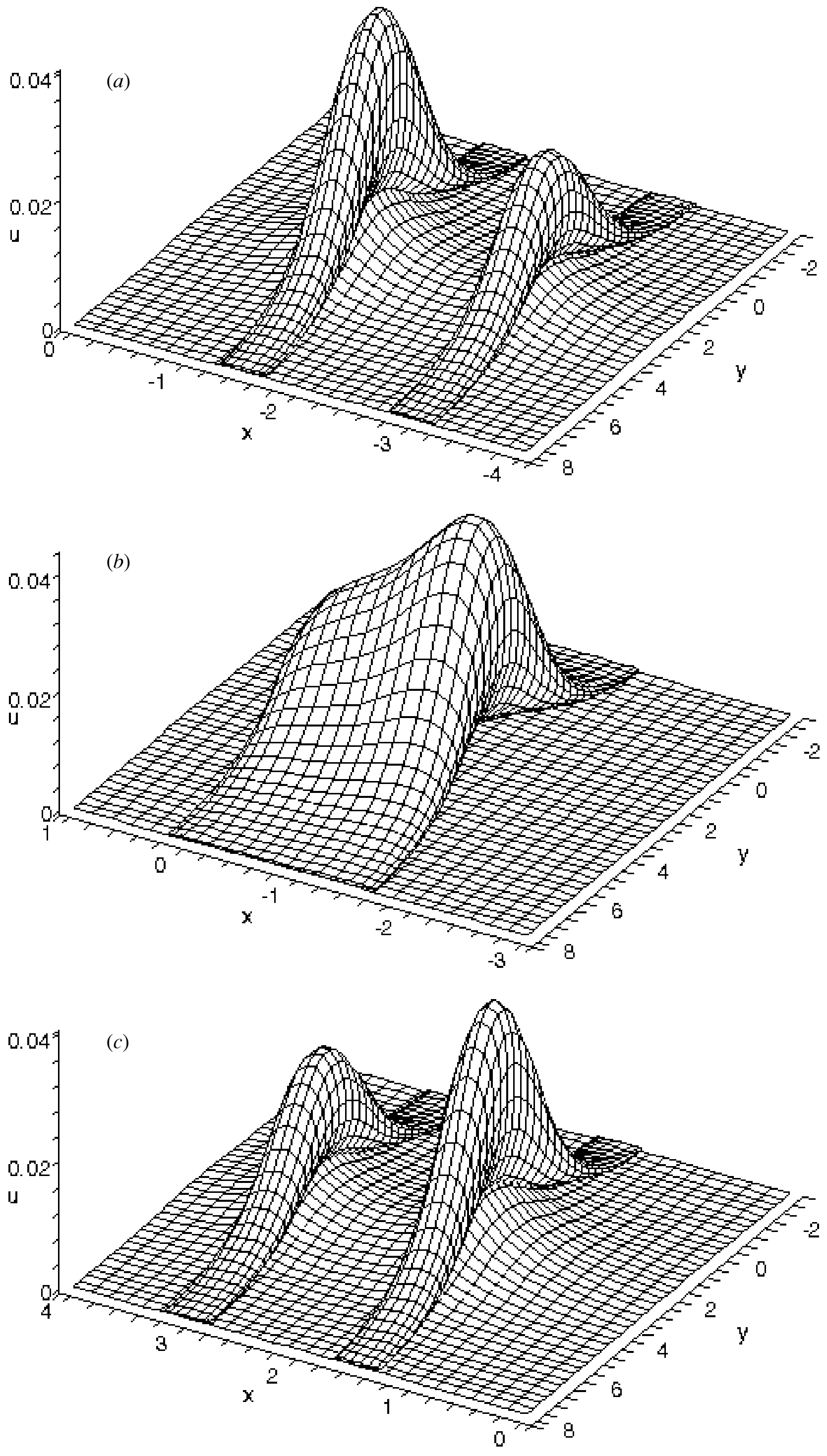

Figure 1. The evolution of the interactions of two semifolded localized structures for the physical quantity u expressed by equation (10a) with conditions (23) and (24) at times (a) $t = -15$, (b) $t = -5$ and (c) $t = 15$, respectively.

where M and N are positive integers, then we may construct another type of semifolded localized structures for the physical quantity u. For simplicity, we take

$$p_x = \frac{4}{5}\operatorname{sech}^2(\xi) + \frac{1}{2}\operatorname{sech}^2(\xi - 0.3t), \qquad x = \xi - 1.5\tanh(\xi) - 1.5\tanh(\xi - 0.3t), \tag{27}$$

$$q = \begin{cases} 0, & y \leqslant -\frac{\pi}{2} \\ \sin(y) + 1, & -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2} \\ 2 & y > \frac{\pi}{2}. \end{cases} \tag{28}$$

Then we derive a combined localized coherent structure depicted in figure 2.

According to the above ideas, if we take p and q to have the following forms:

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{29}$$

$$q = \begin{cases} \sum_{j=1}^{N} e_j \exp(n_j y + w_j t + y_{0j}), & n_j y + w_j t + y_{0j} \leqslant 0 \\ \sum_{j=1}^{N} (-e_j \exp(-n_j y - w_j t - y_{0j}) + 2e_j), & n_j y + w_j t + y_{0j} > 0, \end{cases} \tag{30}$$

where M and N are positive integers, then we may construct third-type semifolded localized structures for the physical quantity u. For convenience, we select

$$p_x = \frac{4}{5}\operatorname{sech}^2(\xi) + \frac{1}{2}\operatorname{sech}^2(\xi - 0.3t), \qquad x = \xi - 1.5\tanh(\xi) - 1.5\tanh(\xi - 0.3t), \tag{31}$$

$$q = \begin{cases} \exp(y) & y \leqslant 0 \\ -\exp(-y) & y > 0, \end{cases} \tag{32}$$

and find that their interaction is also completely elastic. The corresponding plot is depicted in figure 3.

3.3. Non-completely elastic interactions

It is interesting to mention that though the above choices lead to completely elastic interaction behaviours for the (2+1)-dimensional solutions, one can also derive some combined localized coherent structures with non-completely elastic interaction behaviours by selecting p and q appropriately. One of the simple choices of the combined localized coherent structures with non-completely elastic interaction behaviour is

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{33}$$

$$q = a_0 + \sum_{j=1}^{N} B_j \tanh[K_j(y + \beta t) + y_{0j}], \tag{34}$$

where a_0, B_j, K_j, β, w_i and y_{0j} are all arbitrary constants, and M, N are positive integers. We can find that the interaction between semifoldon and dromion may exhibit a novel property, which is non-completely elastic since their shapes are not completely preserved after interaction. In order to clarify this phenomenon more clearly and visually, an example is depicted in figure 4 when the related functions are fixed as follows:

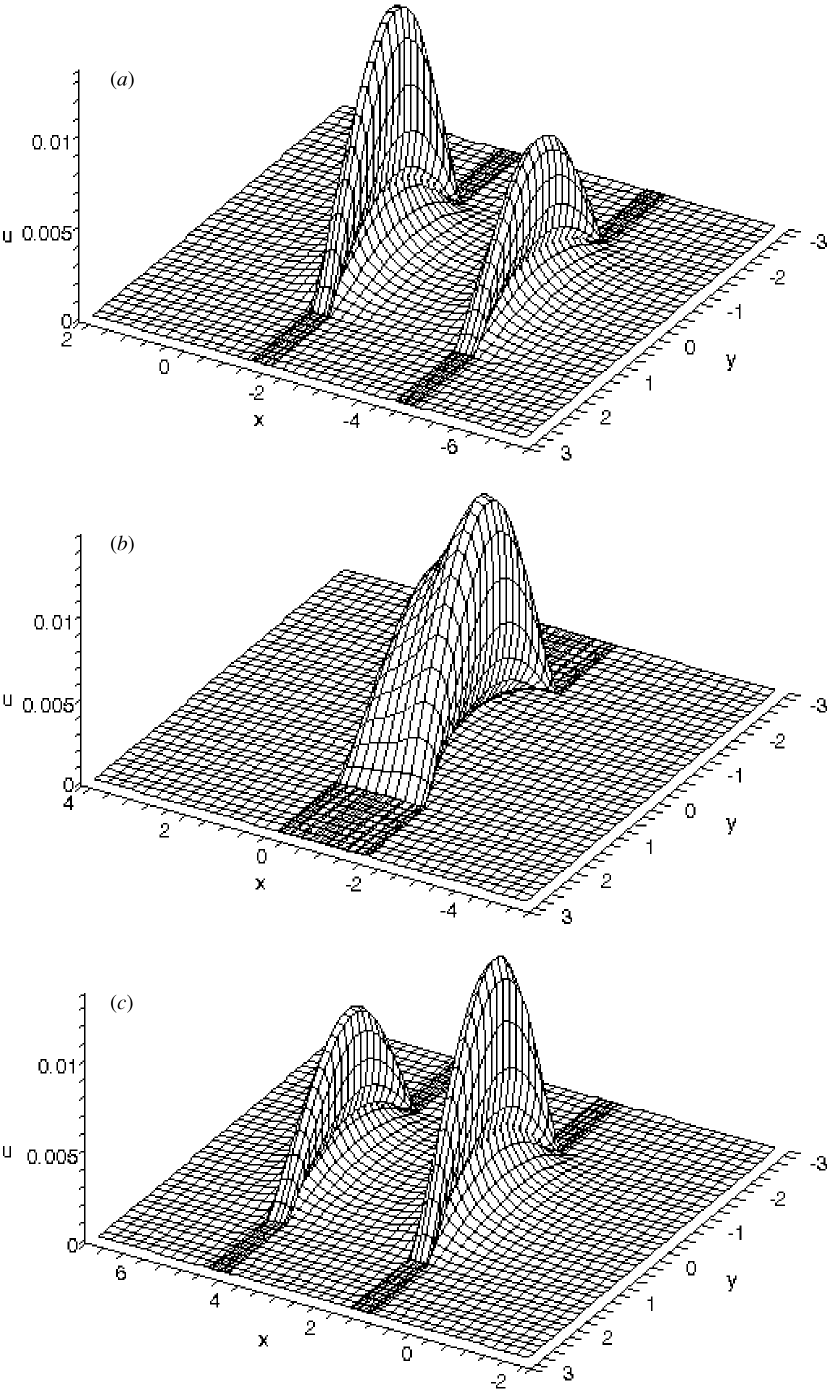

Figure 2. The evolution of the interactions of two semifolded localized structures for the physical quantity u expressed by equation (10a) with conditions (27) and (28) at times (a) $t = -20$, (b) $t = -5$ and (c) $t = 20$, respectively.

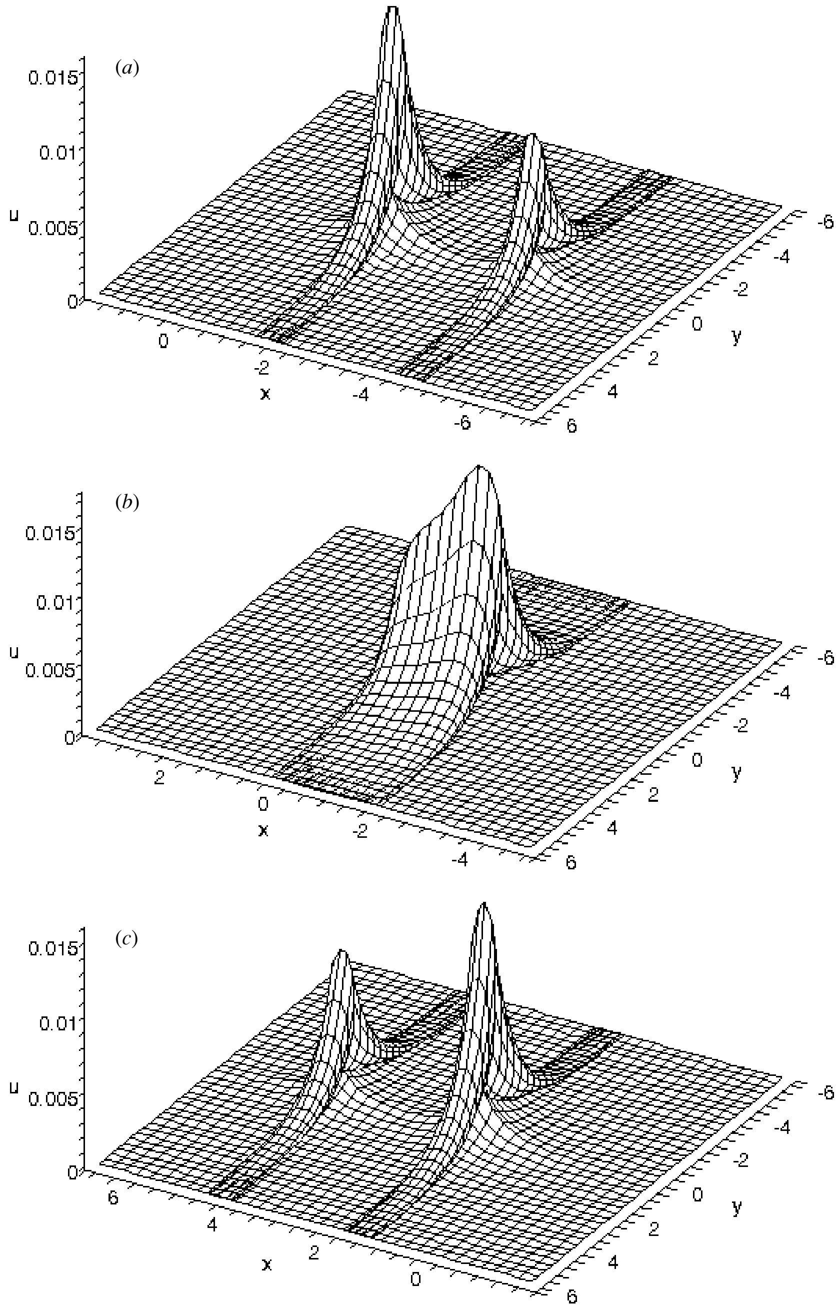

Figure 3. The temporal evolution of two semifolded localized structures interaction for the physical quantity u expressed by equation (10a) with conditions (31) and (32) at times (a) $t = -20$, (b) $t = -5$ and (c) $t = 20$, respectively.

$$p_x = \frac{4}{5}\operatorname{sech}^2(\xi) + \frac{1}{2}\operatorname{sech}^2(\xi - 0.3t),\qquad x = \xi - 1.5\tanh(\xi - 0.3t), \tag{35}$$

$$q = \tanh(y). \tag{36}$$

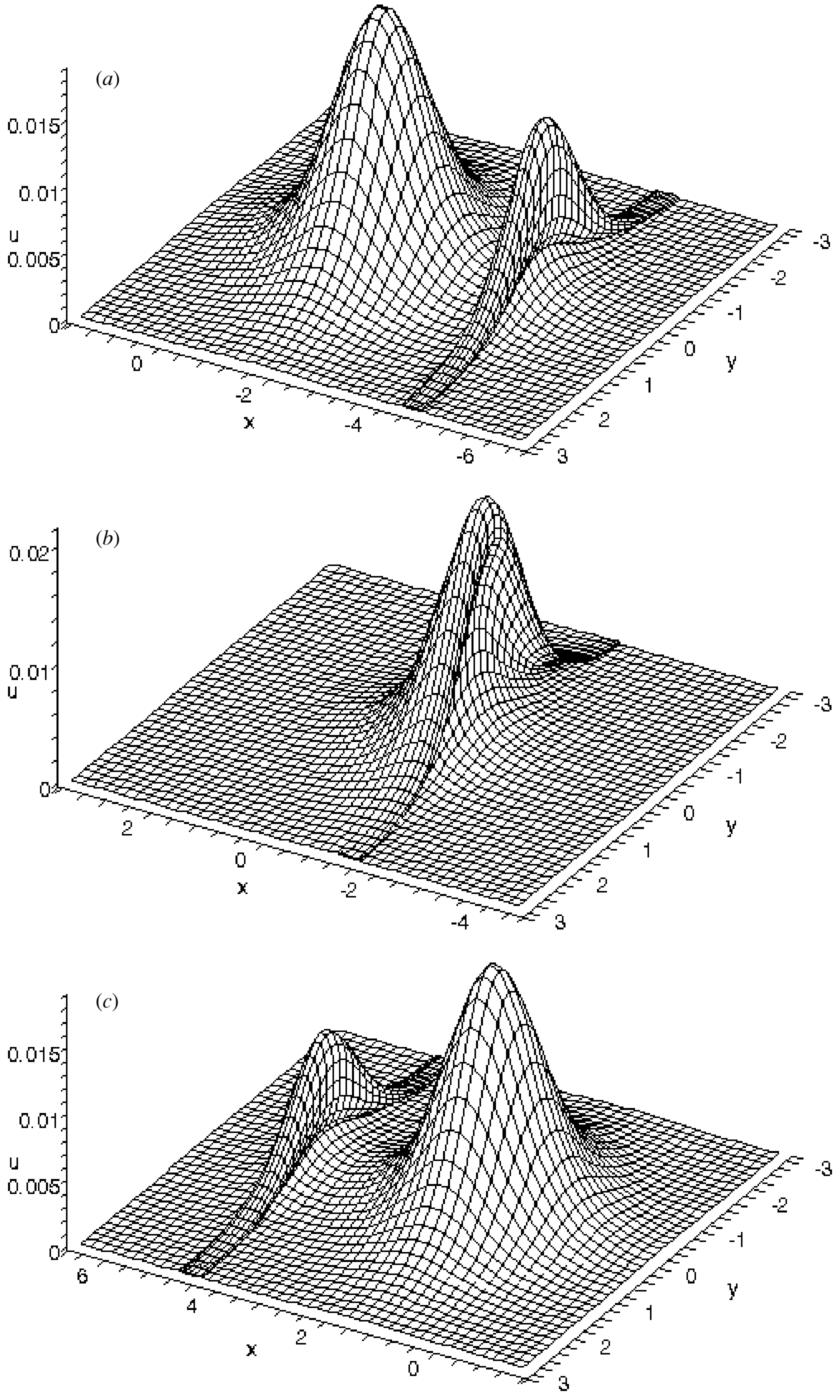

Figure 4. The time evolution of the interaction between semifoldon and dromion as seen in the physical quantity u with conditions (35) and (36) at the times (*a*) $t = -15$, (*b*) $t = -5$ and (*c*) $t = 15$, respectively.

Another example is provided by a combined semifoldon and compacton soliton solutions in the (2+1)-dimensional system. The corresponding ansatz is

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{37}$$

$$q = \begin{cases} a_0, & y + \beta_i t \leqslant y_{0i} - \frac{\pi}{2k_i} \\ a_0 + \sum_{i=1}^{N} (b_i \sin(k_i(y + \beta_i t - y_{0i})) + b_i), & y_{0i} - \frac{\pi}{2k_i} < y + \beta_i t \leqslant y_{0i} + \frac{\pi}{2k_i} \\ a_0 + \sum_{i=1}^{M} 2b_i, & y + \beta_i t > y_{0i} + \frac{\pi}{2k_i}, \end{cases} \tag{38}$$

where M and N are positive integers, then we may construct another non-completely elastic interaction example for the physical quantity u. For simplicity, we can choose

$$p_x = \tfrac{4}{5}\,\text{sech}^2(\xi) + \tfrac{1}{2}\,\text{sech}^2(\xi - 0.3t), \qquad x = \xi - 1.5\tanh(\xi - 0.3t), \tag{39}$$

$$q = \begin{cases} 0, & y \leqslant -\frac{\pi}{2} \\ \sin(y) + 1, & -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2} \\ 2 & y > \frac{\pi}{2}, \end{cases} \tag{40}$$

and obtain a combined semifoldon–compacton localized coherent structure with non-completely elastic interaction behaviour as displayed in figure 5.

In fact, we can also construct combined semifoldon–peakon localized coherent structures with non-completely elastic interaction behaviours by selecting p and q as

$$p_x = \sum_{i=1}^{M} U_i(\xi + w_i t), \qquad x = \xi + \sum_{i=1}^{M} X_i(\xi + w_i t), \tag{41}$$

$$q = \begin{cases} \sum_{j=1}^{N} e_j \exp(n_j y + w_j t + y_{0j}), & n_j y + w_j t + y_{0j} \leqslant 0 \\ \sum_{j=1}^{N} (-e_j \exp(-n_j y - w_j t - y_{0j}) + 2e_j), & n_j y + w_j t + y_{0j} > 0, \end{cases} \tag{42}$$

where M and N are positive integers. Because of the complexity, here we just write down the simplest case

$$p_x = \tfrac{4}{5}\,\text{sech}^2(\xi) + \tfrac{1}{2}\,\text{sech}^2(\xi - 0.3t), \qquad x = \xi - 1.5\tanh(\xi - 0.3t), \tag{43}$$

$$q = \begin{cases} \exp(y) & y \leqslant 0 \\ -\exp(-y) & y > 0. \end{cases} \tag{44}$$

The corresponding time evolution plot is displayed in figure 6.

4. Discussion and summary

Starting from the obtained variable-separated excitations, which describe some quite universal (2+1)-dimensional physical models of a (2+1)-dimensional system, we discuss the interactions among semifoldons, peakons, dromions and compactons both analytically and graphically, and reveal some novel properties and interesting behaviours: the interactions among semifoldons are completely elastic and possess phase shifts, and the interactions of semifoldon–dromion, semifoldon–compacton and semifoldon–peakon are non-completely elastic depending on the

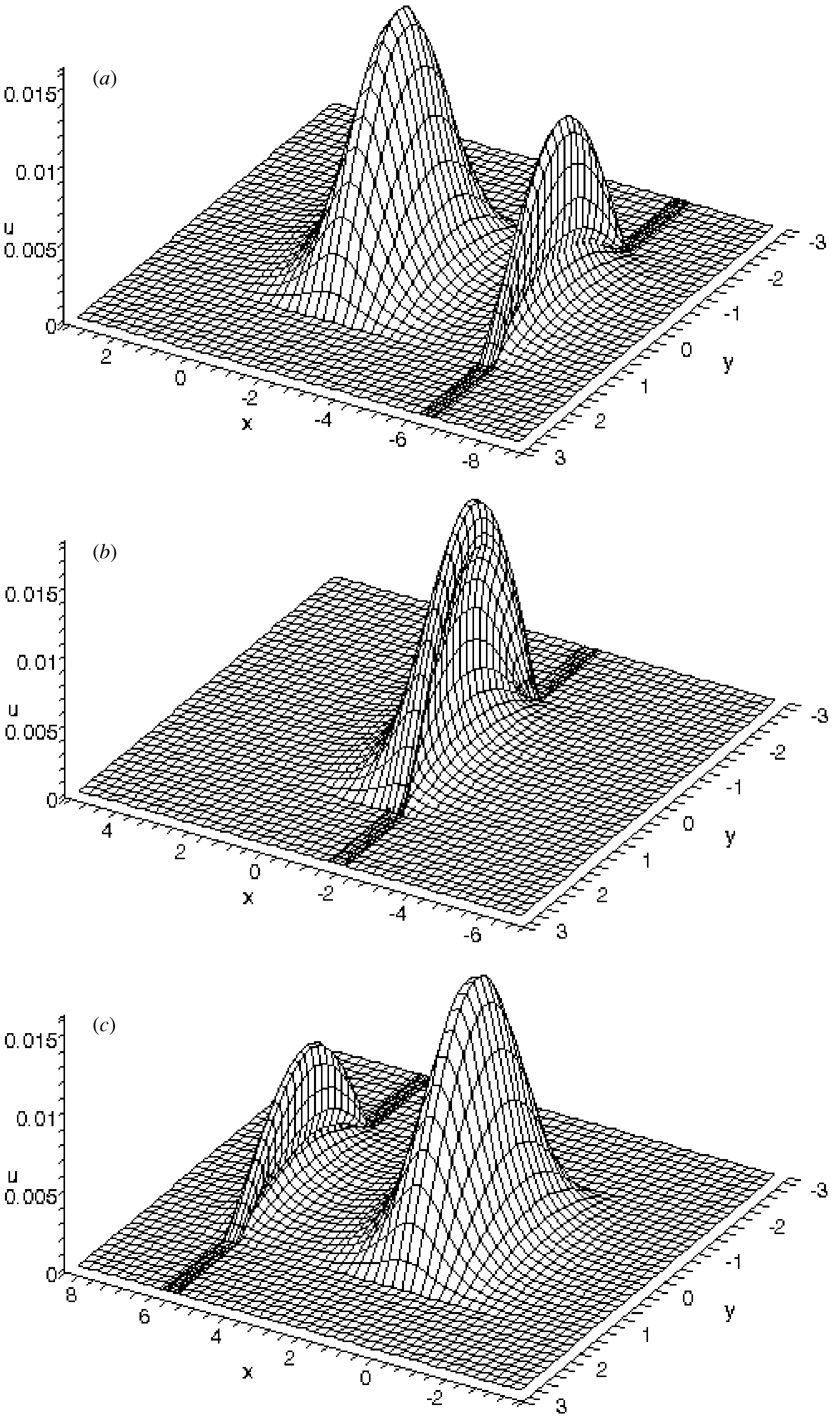

Figure 5. The evolution of the interaction between semifoldon and compacton for the physical quantity u expressed by equation (10a) with conditions (39) and (40) at times (a) $t = -20$, (b) $t = -5$ and (c) $t = 20$, respectively.

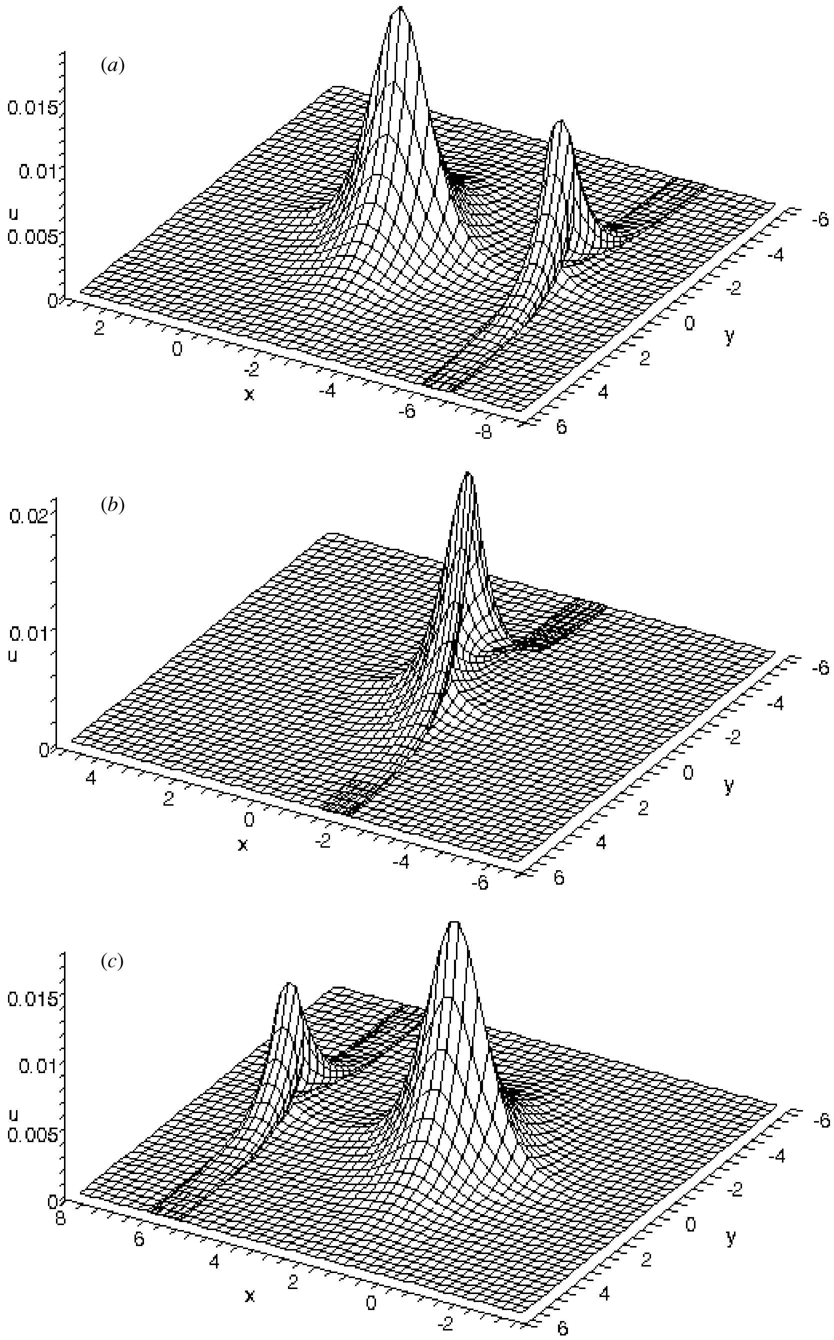

Figure 6. The evolution of the interaction between semifoldon and peakon for the physical quantity u expressed by equation (10*a*) with conditions (43) and (44) at times (*a*) $t = -20$, (*b*) $t = -5$ and (*c*) $t = 20$, respectively.

specific details of the solutions. To our knowledge, the semifolded solitary wave and/or semifoldon excitations for the (2+1)-dimensional GNNV system have not been reported previously in the literature.

In [10], the author pointed out that the localized solutions of the DS equation, say, dromions, can be remote controlled by choosing a suitable motion of the boundaries. In [4], Tang and Lou also pointed out that though the localized excitations such as the dromions, lumps, ring solitons, peakons and foldons possess zero boundary conditions for the quantity u, the boundary conditions for other quantities, say, the mean flow for the DS model, are not identically zero. The different selections of the arbitrary functions p and q in (10a) correspond to the different selections of the boundary conditions of those fields (or potentials) with nonzero boundary conditions and vice versa. That means, in some sense, the foldons, semifoldons and other types of localized excitations for some physical quantities are remote controlled by some other quantities (or potentials). This fact hints that it is possible for one to observe the foldons, semifolons and other types of localized excitations from the systems governed by the VSA on EHBM solvable models by inputting suitable boundary conditions. For foldons, the input boundaries may be selected as (1+1)-dimensional loop solitons.

Because of the complexity of semifolded phenomena and the wide applications of the soliton theory, what more we can learn about the new localized structures and interactions between different types of solitary waves and their applications in reality is worth further study.

Acknowledgments

The authors would like to express their sincere thanks to referees for their helpful advice and suggestions. The project was supported by the National Natural Science Foundation of China and the Natural Science Foundation of Shandong Province in China.

References

[1] Bullough R K and Caudrey P J 1980 *Solitons* (Berlin: Springer)
Dodd R K, Eilbeck J C and Morris H C 1984 *Solitons and Nonlinear Equations* (London: Academic)
Drazin P G and Johnson R S 1988 *Solitons: An Introduction* (Cambridge: Cambridge University Press)
Gu C H 1995 *Soliton Theory and Its Applications* (New York: Springer)
Infeld E and Rowlands G 2000 *Nonlinear Waves, Solitons, and Chaos* 2nd edn (Cambridge: Cambridge University Press)
Rogers C and Schief W K 2002 Bäcklund and Darboux transformations, geometry and modern applications in soliton theory *Cambridge Texts in Applied Mathematics* (Cambridge: Cambridge University Press)
[2] Trewick S C, Henshaw T F, Hausinger R P, Lindahl T and Sedgwick B 2002 *Nature* **419** 174
Lockless S W and Ranganathan R 1999 *Science* **286** 295
Lindgard P A and Bohr H 1996 *Phys. Rev. Lett.* **77** 779
[3] Goodman M B, Ernstrom G G, Chelur D S, Hagan R O, Yao C A and Chalfie M 2002 *Nature* **415** 1039
MacInnis B L and Campenot R B 2002 *Science* **295** 1536
[4] Tang X Y, Lou S Y and Zhang Y 2002 *Phys. Rev.* E **65** 46601
Tang X Y, Chen C L and Lou S Y 2002 *J. Phys. A: Math. Gen.* **35** L293
[5] Bai C L and Zhao H 2004 *Z. Naturfursch* A **59** 412
Bai C L and Zhao H 2004 *Eur. Phys. J.* B **42** 581
[6] Bai C L and Zhao H 2005 *Chin. Phys.* **14** 285
Lou S Y 2002 *J. Phys. A: Math. Gen.* **35** 10619
[7] Boiti M, Leon J J P, Manna M and Pempinelli F 1986 *Inverse Problems* **2** 271
[8] Radha R and Lakshmanan M 1994 *J. Math. Phys.* **35** 4746
[9] Bai C L 2004 *J. Phys. Soc. Japan* **73** 37
Bai C L and Zhao H 2005 *Chaos Soliton Fractals* **23** 777
[10] Fokas A S and Santini P M 1989 *Phys. Rev. Lett.* **63** 1329
[11] Lou S Y 2003 *J. Phys. A: Math. Gen.* **36** 3877
[12] Bai C L, Hu H Q, Wang W J and Zhao H 2004 *Z. Naturfursch* A **59** 729
Bai C L, Bai C J and Zhao H 2005 *Chaos Soliton Fractals* **25** 481